Homework 10.

A simple magnifier: compensation for chromatic aberrations.

In an optical system a simple magnifier is used to aid the eye in viewing images formed by prior components of the system. Such magnifiers are called *oculars* or *eyepieces*. The image, formed by the primary component of the optical system, say, the microscope objective, serves as the object viewed by the eyepiece. The angular magnification of the objective contributes to the total magnification of the optical system.

To form high quality "crisp" image, the ocular has to be free as far as possible from all kinds of aberrations which lead to image distortion. One type of aberrations is called "chromatic aberration" and the reason for this is the dispersion - dependence of the refractive index of the lens material on the frequency (i.e. the "color") of the light. If we tale a look at the "lensmake "equation (below):

$$\frac{1}{f} \approx \left(\frac{n_{lens}}{n_{env}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (1)$$

We can see that change in refraction index leads to change in the focal distance of the lens, and, hence to change in the image position. Chromatic aberrations can be seen as a rainbow blur of a sharp edge of the polychromic image.

As we will see below, there is an elegant way to compensate for the chromatic aberrations. This way is based on using an objective consisting of two lenses. To illustrate the operation principle, I introduce a general expression of the focal distance F of a system, consisting two thin lenses with focal lengths f_1 and f_2 separated by a distance L:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} \quad (2)$$

The focal distance is counted from the lenses. We will accept this formula without the proof for now. If L is much less than f_1 and f_2, then expression (2) becomes a familiar formula for the focal distance of a combined lens:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad (3)$$

We are going to see how does the reciprocal focal distance of the two-lens-system depend on the refractive index of the lens material. For this we will simplify the "lensmaker" equation for lens 1 as:

$$\frac{1}{f_1} \approx (n-1)K_1 \quad (4),$$

where n is the refractive index of the lens material, the media is air with $n \approx 1$, and $K_1 \equiv \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Writing the expression for the focal distance of lens 2 in the same manner and plugging these expressions into the expression for the focal lengths of two-lens system (formula2), we obtain:

$$\frac{1}{F} = (n-1)K_1 + (n-1)K_2 - L(n-1)^2 K_1 K_2 \quad (5)$$

The reciprocal focal distance of the two-lens system nonmonotonically on the refractive index of the lend material n. This dependence has maximum. As long as the reciprocal focal distance 1/F is near the maximum, it is not affected by small variations of n, produced by chromatic aberrations. In other words, if we will choose the distance L between the lenses so that we hit the maximum, the two-lens objective will be unsensitive to chromatic aberrations. So, we have to find L corresponding to the maximum in the dependence 1/F as a function of n (expression 5). To do this, we will find the derivative of 1/F with respect of n and make it equal to zero:

$$\frac{d(\frac{1}{F})}{dn} = K_1 + K_2 - 2L(n-1)K_1 K_2 = 0 \quad (6)$$

From here we can find the optimal distance L_O:

$$L_O = \frac{1}{2}\left[\frac{1}{K_1(n-1)} + \frac{1}{K_2(n-1)}\right] \quad (7)$$

But, taking into account expression (4), we can transform (7) into:

$$L_O = \frac{1}{2}[f_1 + f_2] \quad (8)$$

So, placing the lenses at a distance equal to the arithmetic mean of their focal lengths, we can effectively compensate for chromatic aberrations. The condition expressed in equation (8) is valid for any lens shape, so the lens shape is left as a latitude to compensate for other aberrations.

Problems:

1. An eyepiece consists oif two lenses having focal lengths of 625cm and 2.5 cm. Determine their optimal separation in reducing chromatic aberration, their equivalent focal length and their angular magnification when viewing an image at infinity.

2. A magnifier is made of two thin plano-convex lenses each of 3-cm focal length and spaced 2.8 cm apart. Find equivalent focal length and magnifying power for an image formed at the near point of the eye.