

Homework 10.

Ohm's law. Ideal and real wires. Parallel and series connections.

Just to remind: voltage (or, more correct, *potential difference*) between two points is the work which we have to do to move a unit positive charge from point 1 to point 2 or the work which the electric force does when a unit positive charge is moved from point 2 to point 1. ***This work does not depend on the path we have chosen to move the charge.*** The voltage is measured in joules per coulomb, or *volts*. The potential difference of 5V (5 volts) between point 1 and point 2 (potential in point 2 minus potential in point 1 !) means that the electric force will perform work of 5J on 1C which is moved from point 2 to point 1. If potential is higher in the point 2 then the potential difference is positive, otherwise it is negative. If we maintain a positive potential difference between two points and connect these two points with a piece of metal wire or other material which is able to conduct electrical current the charged particles in the wire will move from point 2 to point 1 if they are positive, and from point 1 to point 2 if they are negative, so there will be electrical current in the wire. The current I between two points is proportional to the voltage U between these points.

$$U = R \cdot I \quad (1)$$

The coefficient R is called electrical *resistance*. The resistance of a piece of wire depends on the length of the wire, on its thickness and on the wire material.

The resistance is measured in Ohms. This unit is named after Georg Simon Ohm, German physicist and high school teacher.



Georg Simon Ohm (1789_1854)

The resistance of 1 Ohm means that if we apply a potential difference of 1 Volt we will obtain current of 1A. The magnitude, which is reciprocal to the resistance is called conductance. The unit of conductance is 1/Ohm, or Siemens (S).

Using my favorite analogue between the current flow in a conducting wire and water flow in a tube, I can represent the resistance as the magnitude, proportional to the inverse diameter of the tube. A wider tube allows more water to flow through it, so it's "resistance" is low.

Resistance R of a wire can be calculated using the following expression:

$$R = \rho \frac{l}{A}, \quad (2)$$

where l is the length of the wire, A is the cross-section area of the wire and ρ is the coefficient which is called *resistivity*. Resistivity is measured in Ohm/m and does not depend on the wire geometry and is a property of the wire material. Parameter σ , reciprocal to resistivity is called "*conductivity*":

$$\sigma = \frac{1}{\rho} \quad (3)$$

All materials follow into three major groups depending on their conductivity (or resistivity), (Table 1). Materials with a very low conductivity are called “*insulators*”. They are marked by dots in the left side of Table 1. Examples of insulators are glass, diamond and fused quartz. Highly conductive materials are shown in the left side of the table. These are called “*conductors*” and are mostly metals such as silver, copper and platinum. The conductivity of the third group, *semiconductors*, can be varied in a wide range. That is why it is shown by lines rather than dots in Table 1. All electronics devices such as cell phones, computers, TVs are based on semiconductors (silicon is the most common).

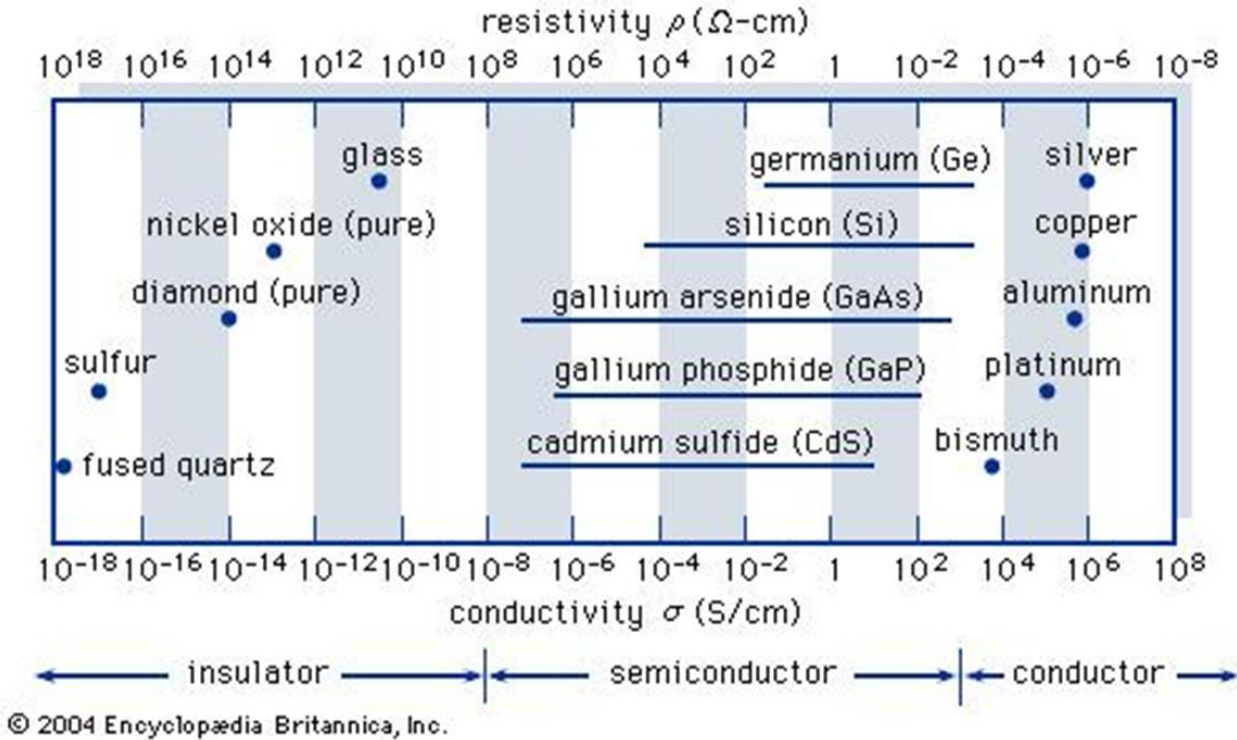


Table 1. Conductivity and resistivity of various materials.

We learned that most of the real wires have finite resistance (the exception is the superconducting wire, but we will not discuss it here now). We will be using 2 types of wire. One is the ideal wire. We will denote in as a smooth line or straight line. The other type is the real wire, which we will denote as a wiggly line (Figure 1).



Figure 1. Ideal and real wires.

The ideal wire has zero resistance. If we will calculate the potential difference (voltage) between 2 points, connected with the ideal wire, we will always obtain zero. It follows from the Ohm’s law:

$$U = I \cdot R = I \cdot 0 = 0 \quad (1).$$

So, if two points are connected by the ideal wire, the voltage between these points is always zero. A piece of real wire we will call “resistor” since it has a finite resistance. We discussed 2 most common ways to connect resistors: *series and parallel*.

Series connection means that 2 or more resistors are connected chainlike, similar to the beads on the thread (Figure 2a). “Parallel connection” means that 2 or more resistors are connected to the same pair of points (Figure 2b).

Resistors, connected in series, share same current. It follows from the charge conservation law.

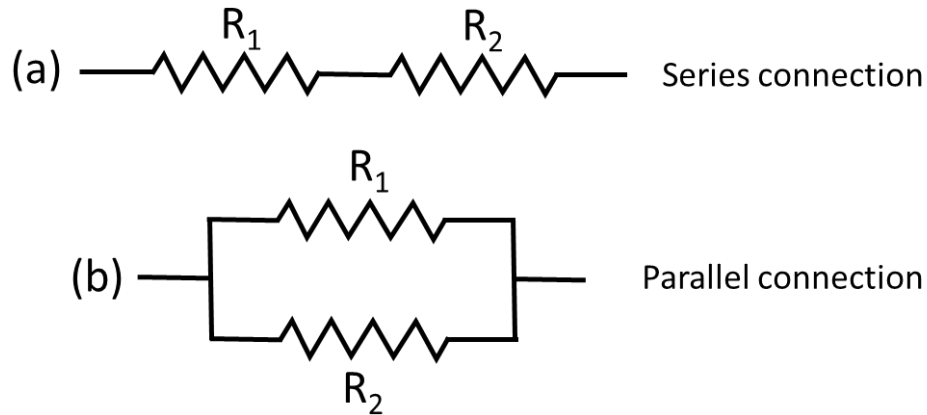


Figure 2. Parallel and series connection.

If a steady current which flows into resistor R_1 (Figure 3a) would have exceeded the current which flows from resistor R_1 into resistor R_2 , then the charge in resistor R_1 would be growing continuously, which is not possible at the steady state conditions.

The total voltage drop across the chain of resistors connected in series is equal to the sum of the voltages at each resistor. So, we have:

$$U_{total} = U_1 + U_2 = IR_1 + IR_2 = I(R_1 + R_2) = I \cdot R_{total} \quad (2)$$

So, for parallel connection $R_{total} = R_1 + R_2$.

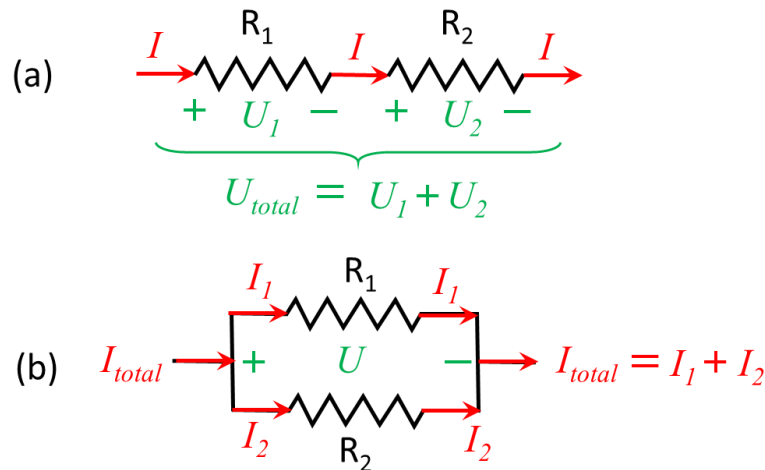


Figure 3. Series and parallel connection of resistors.

For parallel connection voltage drop across all the resistors is the same, but total current is equal to the sum of the currents flowing through each resistor (Figure 3b):

$$I_{total} = I_1 + I_2 = \frac{U}{R_1} + \frac{U}{R_2} = U \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{U}{R_{total}} \quad (3)$$

So, for series connection: $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$. Or, if we have just 2 resistors, connected in parallel:

$$R_{total} = \frac{R_1 \cdot R_2}{R_1 + R_2} \quad (4)$$

We can better understand the difference between parallel and series connections from the picture below:

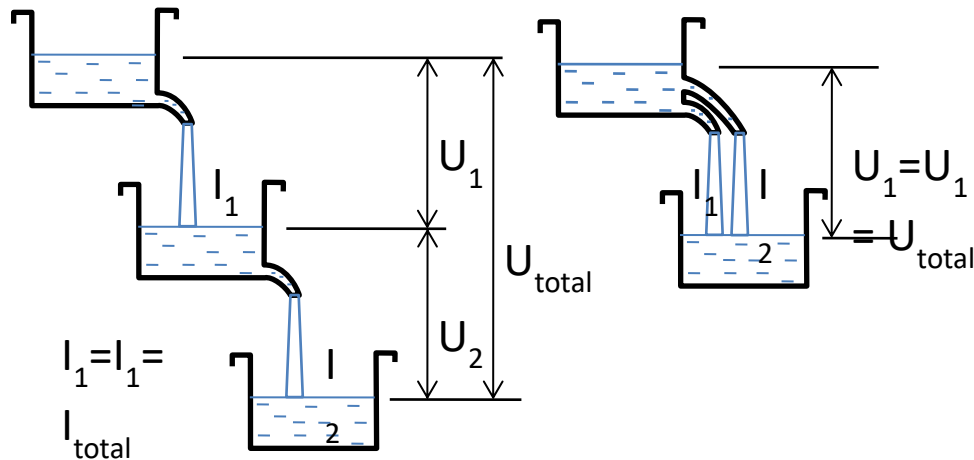


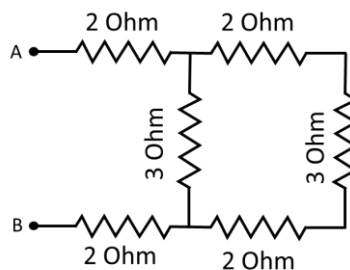
Figure 4. Parallel and series connection as the waterfall cascades.

Series connection: $U_{total} = U_1 + U_2$, $I_{total} = I_1 = I_2$, which means same current, voltages are added

Parallel connection: $U_{total} = U_1 = U_2$, $I_{total} = I_1 + I_2$, which means same voltage, currents are added

Problems:

1. Two conductors are connected in series to a voltage source. Resistance of one conductor is 3 times higher than the resistance of the other. How much times will the total current in the circuit change if we will connect these conductors to the same voltage source in parallel?
2. The resistor network below can be replaced with just 1 resistor, connected between points A and B. Resistance of this resistor is called "equivalent resistance of the network". Find the equivalent resistance:



3. More difficult problem (for volunteers only!): imagine that we infinitely extended the network shown above. Find the equivalent resistance in this case:

