ELASTIC FORCE
DECEMBER 11, 2023

Theory recap

Suppose you take a rubber cord and hold one of its ends in your right hand and the other end in your left hand and start pulling you hands apart. What will happen? You will be able to move your hands away to some distance but then you will starting feeling the resistance of the cord. Why does it happen?

The reason is elasticity. It is the property of objects to resist deformation. When you move your hands apart, the rubber cord is deformed because its length has to change. And it resists being deformed by exerting a force that tries to move your hands back together and bring its length back to normal.

Elasticity is a property of any solid object - rubber, steel, wood, plastic, etc. All of them deform under a force, some less than the others, so some of these materials we usually see as non-deformable. But they only really differ in how big a force is needed to cause the same deformation.

What determines how much does an object deform under a certain force? This was first answered by an English 17-th century scientist Robert Hooke. As an example of an object undergoing deformation we will use a spring. Say it has some equilibrium (when no force is applied to the spring) length l. Then we pull its' end with force F so that it stretches and the length becomes $l+\Delta l$ (therefore the elongation is Δl), F is directly proportional to Δl:

$$F = k\Delta l$$

This relation is called **Hooke's law**. Here k is a coefficient called spring constant - it is a property of a given spring. For example, if a force of 1 N causes some spring to become longer by 1 cm, we know that a force of 2 N will cause this spring to become longer by 2 cm. We can also calculate spring constant for this spring : $k = \dfrac{F}{\Delta l} = \dfrac{1\text{ N}}{1\text{ cm}} = 1\,\dfrac{\text{N}}{\text{cm}}$. We see that spring constant has units $\dfrac{\text{N}}{\text{cm}}$, or in the metric system $\dfrac{\text{N}}{\text{m}}$ which is a slightly less convenient unit for springs we might deal with in everyday life.

Hooke's law explains how a spring scale (also called dynamometer if it is calibrated in Newtons) works. Indeed, if you attach some object of mass m to the spring and let it hang, the spring will become longer proportionally to the mass of the object (technically, to its weight mg, but g is a constant - so we can say that elongation is proportional to m). Then all you have to do is attach a pointer to the end of the spring and calibrate your scale. Masses of 1 kg, 2 kg, 3 kg etc. are equally spaced on this scale because of the Hooke's law.

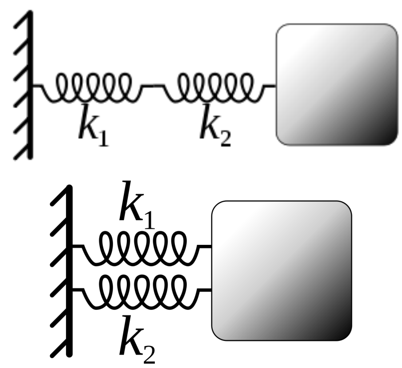
Homework

1. A spring with spring constant $10\ \dfrac{\text{N}}{\text{cm}}$ is attached to the ceiling. We hang a 5 kg block from this spring. Find the deformation of the spring.

2. A dynamometer is calibrated in Newtons. You measure the distance between a 2 N and a 5 N marks and find that it is 6 cm. What is the spring constant of the spring in this dynamometer? What is the distance between a 0 N and a 5 N marks?

3. Amazon continues improving its robots. Now the robot should drag a box by a spring attached to it. The box has mass 10 kg, friction coefficient is 0.5 and spring constant is $20\ \dfrac{\text{N}}{\text{cm}}$. What is deformation of the spring if the robot drags the box with horizontal acceleration 3 m/s^2?

***4.** In systems with several springs a notion of effective spring constant is useful. Suppose that we can replace several springs by one spring and get the same total deformation under the same force. Then spring constant of this one spring is called the effective spring constant of a system. Find effective spring constant for "series" (top figure) and "parallel" (bottom figure) connection of two springs with spring constants $k_1 = 5\ \dfrac{\text{N}}{\text{cm}}$ and $k_2 = 10\ \dfrac{\text{N}}{\text{cm}}$.



Images source: Wikipedia