DISPLACEMENT AT MOTION WITH ACCELERATION

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Theory recap

Recall from the previous class that accelerated motion occurs whenever velocity changes. If velocity changes at a constant rate, this means that acceleration is constant and velocity ($\vec{v}$) dependence on time (t) is very simple:

$$\vec{v}(t) = \vec{v}_0 + \vec{a}\ t$$

Here $\vec{v}_0$ is initial velocity and $\vec{a}$ is acceleration.

The next thing we would like to know is how far do we travel when accelerating constantly. We considered an application of this question: if we want to explore a vertical cave but do not see the bottom and are not sure that our rope will be long enough. To measure the depth of the cave before going in we can throw a rock there and measure the time it takes to reach the bottom (which we would hear). It would be smart to actually "throw" the pebble with zero initial velocity, simply let it go from our hand. Then we need to find how to relate the distance traveled by the pebble to time of travel.

If speed was constant, we would know the answer for distance right away: speed multiplied by time. The problem is that speed is changing. But motion with constant acceleration has a nice feature: speed changes at a constant rate. Because of this, average velocity is an algebraic average of initial velocity and final velocity. For a pebble initially at rest (velocity equals 0) after time t velocity will be gt (we choose the positive direction to be down, it will be more convenient). Algebraic average of initial and final velocity is then

$$v_{avg} = \frac{0 + gt}{2} = \frac{gt}{2}$$

Displacement is equal to average velocity multiplied by time:

$$d = v_{avg}t = \frac{gt}{2}t = \frac{gt^2}{2} \tag{1}$$

Remember that this formula applies for motion with constant acceleration starting at zero velocity. It applies to any kind of motion when the acceleration is constant - you just need to replace g with the corresponding acceleration a. If it took our pebble 3 seconds to fall, we calculate the depth of the cave to be

$$d = \frac{gt^2}{2} = \frac{10 \cdot 3^2}{2}\text{m} = 45\ \text{m}$$

So a standard 50 m rope will be enough.

Homework

1. During the class we have seen a video of an experiment where the time it takes an object to fall from a certain height was measured using an acoustic stopwatch. From the table below with the experimentally measured values, find the free fall acceleration

for each of the 3 heights, and then find the average. The formula you can use comes from a simple inversion of the formula (1): $g = \frac{2d}{t^2}$.

d, m	0.66	1.01	1.43
t, s	0.381	0.487	0.547

2. Let us find how long the runway in an airport should be so that an airplane has enough space to gain speed. An airplane initially at rest accelerates with constant acceleration $a = 2$ m/s^2 until it gets to the takeoff speed of $v = 80$ m/s.

(a) What time does it take the airplane to reach the takeoff speed?

(b) How far does the airplane move before taking off? (normally you would want a runway to be about twice as long - to allow some space for braking in case of emergency).

(c) What is average velocity of the airplane during acceleration?

3. A coin is falling down for 3 seconds. Initial velocity of the coin is 0. Find the displacement of the coin during the third second.