DISPLACEMENT AT MOTION WITH ACCELERATION

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Theory recap

Last week we derived a formula for the distance d that an object moves after certain time t when moving with acceleration a without any initial velocity:

$$d = \frac{at^2}{2}$$

Now assume that an object moves with acceleration $\vec{a}$ while having a non-zero initial velocity $\vec{v_0}$. We will now derive the general formula for displacement of the object after time t. After time t velocity will be $\vec{v_0} + \vec{a}t$, so average velocity is

$$\vec{v}_{avg} = \frac{\vec{v_0} + \vec{v_0} + \vec{a}t}{2} = \vec{v_0} + \frac{\vec{a}t}{2}$$

Then displacement is

$$\vec{d} = \vec{v}_{avg}t = \vec{v_0}t + \frac{\vec{a}t}{2}t = \vec{v_0}t + \frac{\vec{a}t^2}{2}$$

The main formula to use when finding how far will an object travel during time t while moving with acceleration $\vec{a}$ and starting with initial velocity $\vec{v}_0$ is

$$\vec{d} = \vec{v_0}t + \frac{\vec{a}t^2}{2}$$

As always, pay attention to signs when applying this formula. Assign some direction as positive, then if displacement, velocity or acceleration look in the opposite direction, you should take them with a minus sign.

Alternatively, you could find displacement from average velocity. Then you need to remember that for motion with constant acceleration average velocity is the average between initial and final velocity.

Homework

1. Several weeks ago we calculated acceleration of Tesla model S Performance based on the fact that it reaches speed 60 mph from zero initial speed in $t = 2.4$ s to be $a = 11.1$ m/s^2.
   (a) Now find how far does the car travel during this time moving with acceleration.
   (b) We have also found previously that it would take $t = t6.67$ seconds to brake from 60 mph with acceleration 4 m/s^2. Now find the braking distance (how far does the car travel during braking before it stops).

2. You have a bet with your friend that you could throw a ball higher than the roof of your school. Your school is 6 m high. You throw the ball vertically up with initial speed 15 m/s.
   (a) In what time will it reach the highest point? (hint: at highest point the ball has to stop - if it has not stopped yet it would go even higher)
   (b) What height will the ball reach? Did you win the bet?

(c) In what time after you threw the ball will it return to the ground?

The following problem is a bonus problem.

***3.** You stay next to the front door of the first carriage of a train on a train station. The train starts to move with a constant acceleration. You notice that exactly in 3 seconds after the train started moving the front door of the second carriage passes you. How many carriages will move past you in the following 3 seconds? And then in the next 3 seconds after that? Assume that it's a very long train and all the carriages have the same length.