INTERNAL ENERGY AND SPECIFIC HEAT

MARCH 3, 2024

Theory recap

Internal energy, specific heat. Last time we discussed that temperature is a measure of internal kinetic energy of microscopic atoms and molecules inside an object. Actually, internal kinetic energy is not the only kind of internal energy - there is also internal potential energy which we will not discuss in any detail. Together, they comprise **internal energy**.

If we heat an object, we provide it with more energy - in the form of internal energy. We know that as a result its' temperature increases. So an object with higher temperature has more internal energy than the same object but cold (when it comes to the internal kinetic energy only, this is of course just what we discussed before about temperature indicating the average internal kinetic energy).

In other words, if we change temperature of an object by some amount ΔT, we also change its' internal energy. Let us call the change in internal energy ΔE_{int}. What does ΔE_{int} depend upon besides ΔT? Clearly, if an object is larger, its' internal energy is larger, simply because it has more atoms and molecules. A convenient measure of how big an object is is its' mass m. So we arrive to the formula

$$\Delta E_{int} = cm\Delta t.$$

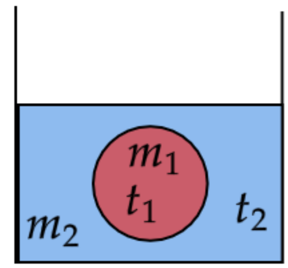
where c is called specific heat capacity, or shorter **specific heat**. It is a property of a given material which can be measured experimentally. Units of specific heat capacity are J/(kg· K) or J/(kg·°C) which are the same, because the change in temperature by 1° C is the same as the change by 1 K. For example, for water $c_{water} = 4200$ J/(kg·K). For iron it is $c_{iron} = 460$ J/(kg ·K). Basically, specific heat capacity tells us how much energy in Joules is needed to heat 1 kg of a substance by 1 K.

Heat transfer How do we change internal energy of an object, for example make it hotter? A common way is heat transfer from another hot object. We know that if two objects of different temperature are brought into contact their temperature will become equal as time passes. For example if you put an apple in the refrigerator the apple will soon become cold, because it is in contact with cold air and surface in the refrigerator. The fact that temperatures of objects in contact become equal is referred to as reaching **thermal equilibrium**[1].

The situation when internal energy is transferred from one object to another is known as heat transfer. In practice we will use the above formula for internal energy change in order to solve various problems. In particular, we can find the equilibrium temperature after two objects of particular temperatures are brought into contact.

[1]Our understanding of temperature would allow us to understand the basic underlying mechanism of reaching thermal equilibrium. Remember that temperature is a measure of internal kinetic energy. So if one object has higher temperature than the other, it means that kinetic energy of atoms and molecules in the first object is larger than in the second object. When these objects are brought into contact, their atoms and molecules collide with each other and during these collisions the ones with higher kinetic energy give their extra energy to the ones with lower kinetic energy. As a result, kinetic energies of atoms and molecules in both objects tend to become the same. Therefore, temperatures tend to become the same.

Let us look at such an example: we take a piece of iron of mass $m_2 = 2$ kg which has temperature $t_2 = 100°$ C and put it in a container with $m_1 = 1$ kg of water at temperature $t_2 = 20°$ C. What temperature will water and iron reach after coming to thermal equilibrium? As often happens in such problems, we will assume that the container with water is well insulated from the environment, so no energy could be transferred between our system (water + iron) and the environment (whatever other objects are around: air, table on which the container sits, etc.).



Let us call the final temperature of water and iron t - that is what we need to find. We would also need specific heat capacities of water and iron which we will call c_1 and c_2 correspondingly (their numerical values were provided above). The most important principle we should use is energy conservation law applied to internal energy. Since the system is insulated and no mechanical energy is dissipated and no work is being done, total internal energy must stay the same. This means that the change in total internal energy is 0:

$$\Delta E_{tot\ int} = \Delta E_1 + \Delta E_2 = 0$$

.

Here ΔE_1 is change of internal energy of water and ΔE_2 is the change of internal energy of iron. Using our general formulas we get:

$$\Delta E_1 = c_1 m_1 \Delta t_1 = c_1 m_1 (t - t_1);\ \Delta E_2 = c_2 m_2 \Delta t_2 = c_2 m_2 (t - t_2)$$

where $\Delta t_1, \Delta t_2$ are changes of temperature of water and iron correspondingly and are found as final temperature minus initial temperature. Plugging this back to our energy conservation condition we get

$$c_1 m_1 (t - t_1) + c_2 m_2 (t - t_2) = 0$$

Now the physics part of the problem has ended and we just need to mathematically solve the equation for t_3. To do it we gather all the terms with the unknown t_3 on the left hand side of the equation and all the terms without t_3 on the right hand side:

$$c_1 m_1 t + c_2 m_2 t = c_1 m_1 t_1 + c_2 m_2 t_2 \Longrightarrow t = \frac{c_1 m_1 t_1 + c_2 m_2 t_2}{c_1 m_1 + c_2 m_2}$$

.

Now we just need to plug in the numbers:

$$t = \frac{4200 \cdot 1 \cdot 20 + 460 \cdot 2 \cdot 100}{460 \cdot 2 + 4200 \cdot 1} °\text{C} = 34°\text{C}$$

.

We see that the answer is 34°C. This is quite close to the initial temperature of the water which means that water temperature did not change much while iron temperature changed a lot. This is because specific heat capacity of water is much bigger than specific heat capacity of iron.

Homework

1. A 500 gram cube of lead is heated from 20°C to 80°C. How much energy was required to heat the lead? The specific heat of lead is 160 J/(kg·°C).

2. Mr. X does not like when his morning coffee is too hot so he adds some cold milk to it. Initially the coffee is at boiling temperature (100°C) and milk is just out of the fridge (10°C). How much milk does Mr. X have to add to 150 g of coffee in order for the mixture to have temperature 65°C? Both coffee and milk have the same specific heat capacity as water.

3. (Bonus) Try to think what are the hottest and the coldest object you have ever seen with your own eyes. You could use Google to find temperatures of different objects, just try to choose a credible source. As an answer to the problem, write down these two objects and their temperatures.

***4.** Some object with initial temperature $t_1 = 100^\circ$ C is put in a glass with water that has temperature $t_2 = 10^\circ$ C. After some time thermal equilibrium established and temperature became $t = 40^\circ$. Then another object, completely the same as the first one and also with initial temperature $t_1 = 100^\circ$ C was put in the same glass with the first object still in it. What will the resulting temperature be?