Homework 16: Coordinate geometry review

HW16 is Due Feb 11.

1. Coordinate geometry: Introduction

In this section of the course, we are going to study coordinate geometry. The basic notion is the **coordinate plane**– a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the x-**axis** and the y-**axis**. x-axis is usually drawn horizontally, and y-axis — vertically. These two axes have a **scale** – "distance" from the origin.
The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point P, draw lines through P perpendicular to the x- and y-axes. These lines intersect the axes in points with coordinates x_0 and y_0. Then the point P has x-coordinate x_0, and y-coordinate y_0, and the notation for that is: $P(x_0, y_0)$.
The **midpoint** M of a segment AB with endpoints A(x_1, y_1) and B(x_2, y_2) has coordinates: $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

2. Lines

Given some relation which involves variables x, y (such as $x + 2y = 0$ or $y = x^2 + 1$), we can plot on the coordinate plane all points $M(x, y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the **graph** of the given relation.

Every relation (**equation**) of the form: $y = \boldsymbol{m}x + \boldsymbol{b}$

where $\boldsymbol{m}, \boldsymbol{b}$ are some numbers, defines a straight line. The slope of this line is determined by m: as you move along the line, y changes m times as fast as x, so if you increase x by 1, then y will increase by m. In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of y: Δ =y_2 −y_1 by the change of x: $\Delta = x_2 - x_1$:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

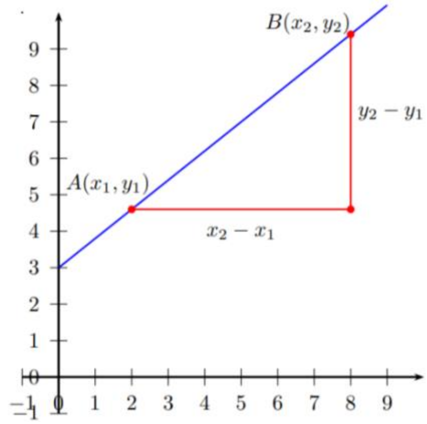


Fig. 1

Two non vertical lines are **parallel** if and only if they have the **same slope**.
In the equation $y = mx + b$, b is a y-**intercept**, and determines where the line intersects the vertical axis (y-axis). The equation of the **vertical** line is $x = k$, and the equation of the **horizontal** line is $y = k$. Notice that in case of the vertical line, the slope is undefined.

3. Distance between points. Circles.

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the following formula:

$$\boldsymbol{d} = \sqrt{(\boldsymbol{x_2} - \boldsymbol{x_2})^2 + (\boldsymbol{y_2} - \boldsymbol{y_1})^2}$$

This formula is a straightforward consequence of the Pythagoras' Theorem (Fig. 1).

The equation of the circle with the center $M(x_0, y_0)$ and radius $\boldsymbol{r}$ is:

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

This equation means, that points (x, y) should be at distance $\boldsymbol{r}$ from the given point $M(x_0, y_0)$.

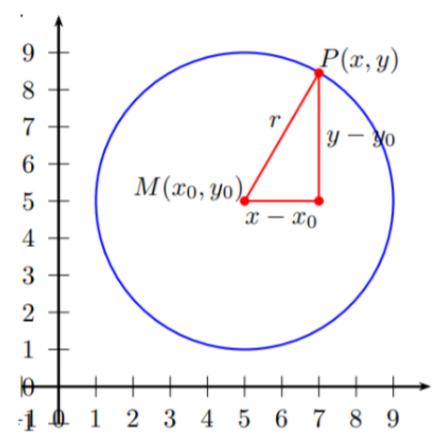


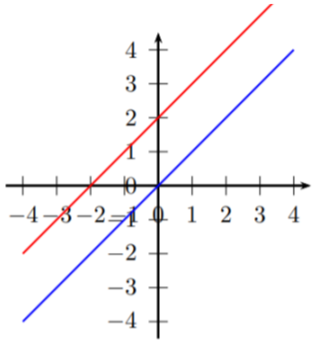
Fig.2

4. Graphs of functions

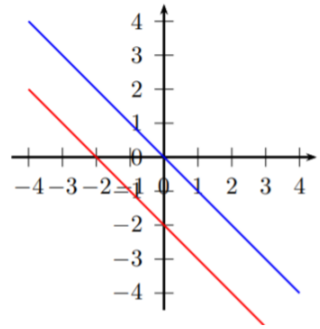
In general, the relation between x and y could be more complicated and could be given by some formula of the form $y = f(x)$, where f is some function of x (i.e., some formula which contains x). Then the set of all points whose coordinates satisfy this relation is called the **graph** of f.

Line. The graph of the function $y = mx + b$ is a straight line. The coefficient $\boldsymbol{m}$ is called the *slope*.

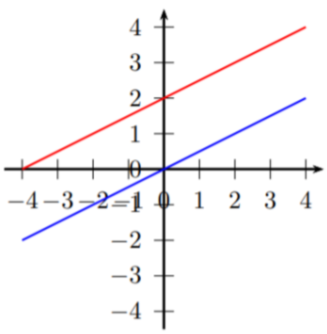
$y = x$; $y = x + 2$:



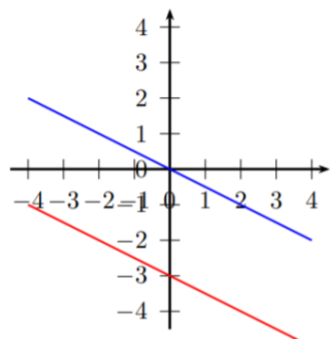
$y = -x$; $y = -x - 2$:



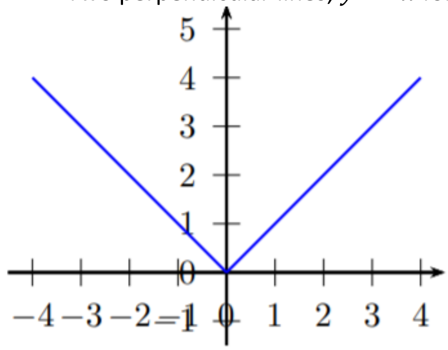
$y = \frac{1}{2}x$; $y = \frac{1}{2}x + 2$:



$y = -\frac{1}{2}x$; $y = -\frac{1}{2}x - 3$:



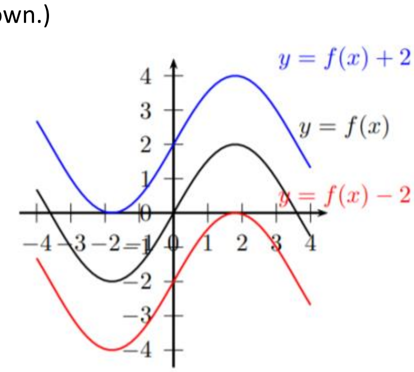
Absolute value of a line. $\boldsymbol{y = |x|}$ Two perpendicular lines, $y = x$ for $x > 0$ and $y = -x$ for $x < 0$.

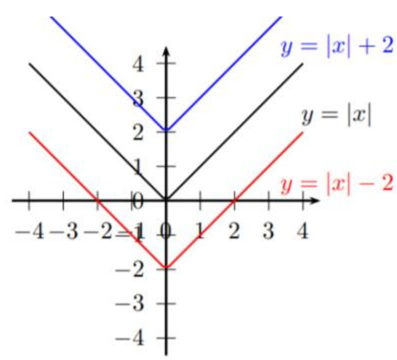


5. Function transformations

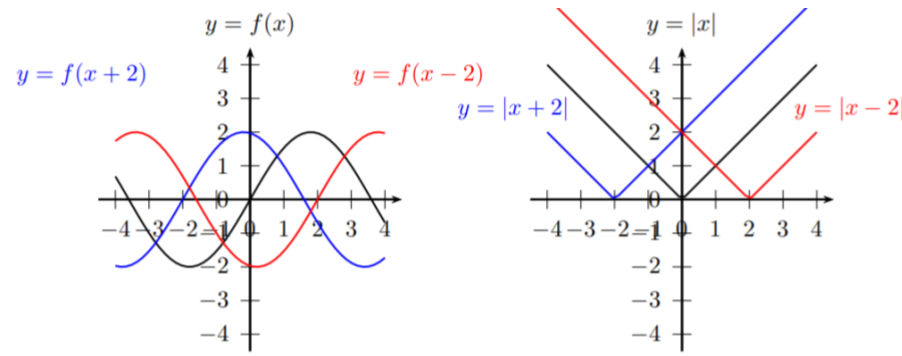
Having learned a number of basic graphs, we can produce new graphs, by doing certain transformations of the equations. Here are two of them.

Vertical translations: Adding constant $\boldsymbol{c}$ to the right-hand side of equation shifts the graph by $\boldsymbol{c}$ units up (if c is positive; if c is negative, it shifts by |c| down.)





Horizontal translations: Adding constant ***c*** to x shifts the graph by ***c*** units left if ***c*** is positive; if ***c*** is negative, it shifts by ***c*** right.



Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

ALL GRAPHS/POINTS/FIGURES SHOULD BE DRAWN BY YOU - NOT PRINTED! USE QUADRILE PAPER!

1. Find the coordinates of the midpoint of the segment AB, where $A = (3, 11)$, $B = (7, 5)$.

2. Draw points $A(4, 1)$, $B(3, 5)$, $C(-1, 4)$. If you did everything correctly, you will get 3 vertices of a square. What are the coordinates of the fourth vertex? What is the area of this square?

3. 3 points $(0, 0)$, $(1, 3)$, $(5, -2)$ are the three vertices of a parallelogram. What are the coordinates of the remaining vertex? (Hints: check the slopes of each line.)

4. Consider the triangle ΔABC with the vertices $A(-2, -1)$, $B(2, 0)$, $C(2, 1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.

5. In this problem you will find equations that describe some lines.
   a. What is the equation whose graph is the $y - axis$?
   b. What is the equation of a line whose points all lie 5 units above the $x - axis$?
   c. Is the graph of $y = x$ a line? Draw it.
   d. Find the equation of a line that contains the points $(1, -1)$, $(2, -2)$, and $(3, -3)$.

6. For each of the equations below, draw the graph, then draw the perpendicular line (going through the point (0, 0)) and then write the equation of the perpendicular line

   a. $y = 3x$ b. $y = -\frac{1}{2}x$

   Can you determine the general rule: if the slope of a line is k, what is the slope of the perpendicular line?
7. Find the equation of the line through $(1, 1)$ with slope 2.
8. Find the equation of the line through points $(1, 1)$ and $(3, 7)$. [Hint: what is the slope?]

9. (a) Find k if $(1, 9)$ is on the graph of $y - 2x = k$. Sketch the graph.
   (b) Find k if $(1, k)$ is on the graph of $5x + 4y - 1 = 0$. Sketch the graph
   A line written in this form, $Ax + Bx + C = 0$, is known as a *standard form*. When this form could be more useful than the slope – intercept form?

10. Find the intersection point of a line $y = x - 3$ and a line $y = -2x + 6$ algebraically solving system equations. Then, sketch the graphs of these lines – did the coordinates of the intersecting point match your solution for x and y?

11. Using the shape of the function y = |x|, sketch (do not graph) on paper the graphs of the following functions:
    a. $y = |x| + 1$
    b. $y = |x + 1|$
    c. $y = |x - 5| - 3$

Note: to graph – means use a few pairs of points to graph the line/curve of the function. Do not use desmos!
To sketch – produce a sketch which approximates main features of the function, using the equation of the function and other properties (e.g., use slope, intercept, shifts, vertex place …)
For this homework, desmos is only to check your results.