Homework 1: Algebraic operations expansion, factorization,
formulas for a fast multiplication

HW1 is Due October 1

Today we reviewed about how to perform basic operations on algebraic expressions, some simple factorization, algebraic identities, and simplifications. The first page contains some of the main things we discussed today. The homework problems will provide more practice.

1. **Exponents Laws**

   If a is a real number, n is a positive integer $\quad a^n = a \times a \times \cdots \times a$

   $$\underbrace{}_{\text{n-times}}$$

   $$a^0 = 1$$

   $$a^m \times a^n = a^{m+n}$$

   $$a^m \div a^n = a^{m-n}$$

   $$(ab)^n = a^n b^n$$

   $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

   $$a^{-n} = \frac{1}{a^n}$$

   $$(a^m)^n = a^{mn}$$

2. **Radicals**

   $$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$
   $$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. **Main Algebraic Identities**

   $$(a+b)^2 = a^2 + 2ab + b^2$$

   $$(a-b)^2 = a^2 - 2ab + b^2$$

   $$a^2 - b^2 = (a-b)(a+b)$$

   Homework problems on next page

Instructions: Please always write solutions on a **separate sheet of paper**. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. The difference between two numbers is $\frac{5}{12}$. If $\frac{3}{4}$ of the larger number is $\frac{3}{8}$ more than $\frac{1}{2}$ of the smaller, find the larger number.

2. Without a calculator, compute

$$19999 \cdot 20001$$

Is there a shorter way of doing it than the straightforward multiplication?

3. Expand the expressions:
   a. $(a^2 + 2a + 1)(a + 1)$
   b. $(2 - 4x)^2$
   c. $(4x - 7y)(4x + 7y)$
   d. $(6x^2 - y)(7x^2 - 2x - 5)$

4. Divide:
   a. $(b^2 - 2b + 1)/(b - 1)$
   b. $x^2 - 2x - yx + 2y / (x-1)$
   c. $(7x^2 - 2xy - 5y^2)/(x - y)$

5. Factor (i.e., write as a product) the following expressions:
   a. $x^2 + 3x^3$
   b. $x^2 - 2x - yx + 2y$
   c. $4x^2 - 4x + 1$
   d. $4x^2 + 16x + 2xy + 8y$
   e. $x^2(x + 4) + 5(x + 4)$
   f. $100x^8y^2 - 16x^4y^6$

   g. $a^2 + 4ab + 4b^2$
   h. $a^2 - 2a + 1$
   i. $x^2 - 7 \quad Hint: 7 = \left(\sqrt{7}\right)^2$

   j. $a^4 - b^4 Hint: a^4 = (a^2)^2$

6. John takes 15 min to walk from school to the bus station. Jim takes 20 min to walk from the school to the bus station. If the difference in their speeds is 2 km/h, how far is the station from the school?

7. Simplify:

$$a) \ \frac{1}{(x + 1)} - \frac{1}{(x - 1)}$$

$$b) \ \left(1 + \frac{1}{x}\right) \div (x + 1)$$

$$c) \ \left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$