September 17, 2023

Geometry.

Baseline revue test. Geometry.

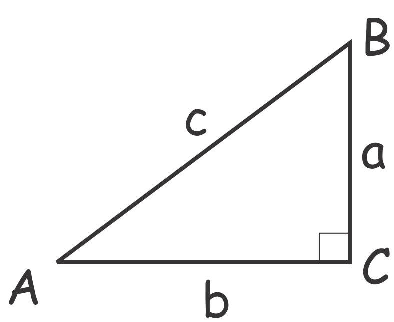
1. List the undefined terms (primitives) of geometry.

2. Give the definition of (i) a segment (ii) a circle.

3. List three congruence tests for triangles.

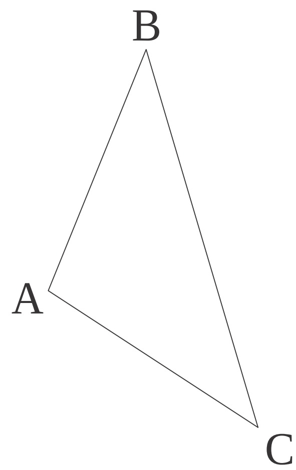
4. State and prove the triangle inequality (hint: this inequality compares the total length of the two sides of a triangle with the length of the third side).

5. State and prove the Pythagorean theorem.

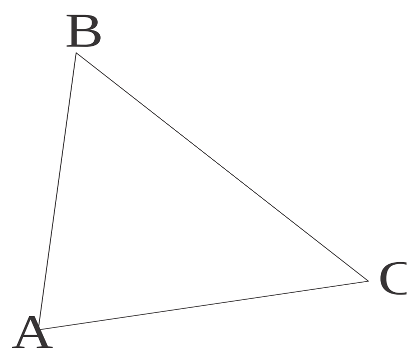


6. List all formulas for the area of a triangle that you know (sides are a, b, c, altitudes to these sides are h_a, h_b, h_c, respectively, the radius of the inscribed circle is r, that of the circumscribed circle is R).

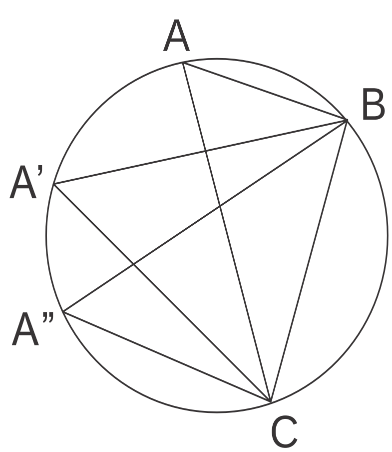
7. Using a compass and a ruler, draw a circle circumscribed around a given triangle.



8. Using a compass and a ruler, draw a circle inscribed into a given triangle.

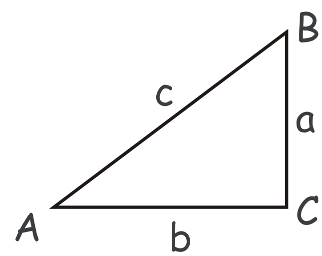


7. Which of the angels $\widehat{BAC}$, $\widehat{BA'C}$, $\widehat{BA''C}$ is the largest? Which is the smallest?

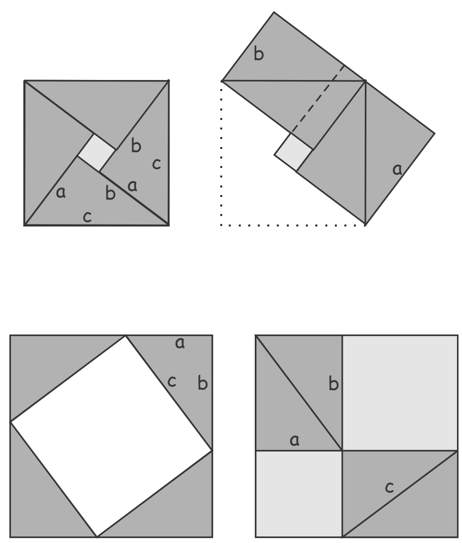


Recap: The Pythagorean Theorem.

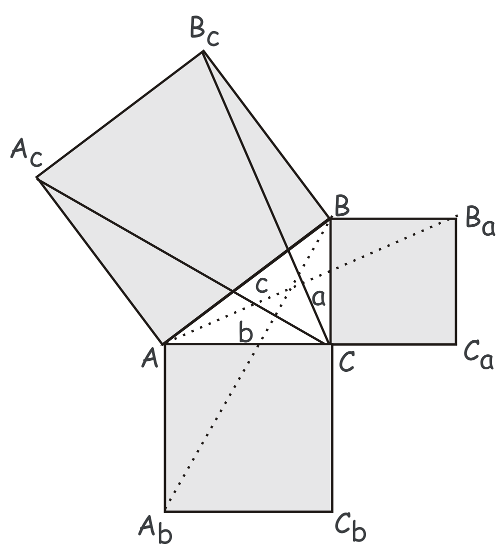
Theorem. In a right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.



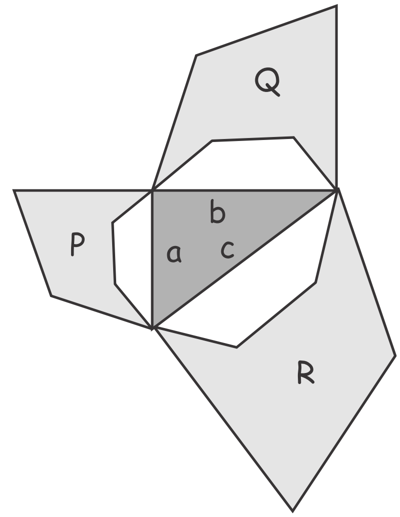
Proof 1. Perhaps, the most elegant are the algebra-free proofs by dissection, as shown in Figures below.



Proof 2. Perhaps, the most famous proof is that by Euclid, although it is neither the simplest, nor the most elegant. It is illustrated in Fig. 3 below.



Generalized Pythagorean Theorem.



If three similar polygons, P, Q and R with areas S_P, S_Q and S_R are constructed on legs a, b and hypotenuse c, respectively, of a right triangle, then,

$$S_P + S_Q = S_R$$

Recap: Inequalities in triangles.

Definition. The angle supplementary to an angle of a triangle is called an exterior angle of this triangle.

Theorem 1. An exterior angle of a triangle is greater than each of the interior angles not supplementary to it.

Theorem 2a. In any triangle,

- the angles opposite to congruent sides are congruent
- the sides opposite to congruent angles are congruent

Theorem 2b. In any triangle,

- The angle opposite to a greater side is greater
- The side opposite to a greater angle is greater

Theorem 3 (triangle inequality). In any triangle, each side is smaller than the sum of the other two sides, and greater than their difference,

$$|AB| - |BC| < |AC| < |AB| + |BC|$$

Theorem 4 (corollary). The line segment connecting any two points is smaller than any broken line connecting these points.

Recap: Parallelogram. Central Symmetry.

Definition. A quadrilateral whose opposite sides are pairwise parallel is called a parallelogram.

Theorem 1a. In a parallelogram, opposite sides are congruent.

Theorem 1b. In a quadrilateral, if the opposite sides are congruent, then this quadrilateral is a parallelogram.

Theorem 1c. In a quadrilateral, if two opposite sides are parallel and congruent, then this quadrilateral is a parallelogram.

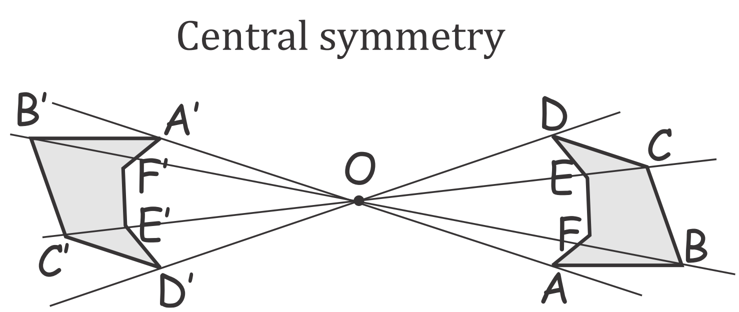
Theorem 2a. In a parallelogram, opposite angles are congruent.

Theorem 2b. In a quadrilateral, if opposite angles are congruent, then this quadrilateral is a parallelogram.

Theorem 3a. In a parallelogram, diagonals bisect each other.

Theorem 3b. In a quadrilateral, if the diagonals bisect each other, then this quadrilateral is a parallelogram.

<u>Recap: Central Symmetry.</u>



Definition. Two points A and A' are symmetric with respect to a point O, if O is the midpoint of the segment AA'.

Definition. Two figures are symmetric with respect to a point O, if for each point of one figure there is a symmetric point belonging to the other figure, and vice versa. The point O is called the center of symmetry.

Symmetric figures are congruent and can be made to coincide by a 180 degree rotation of one of the figures around the center of symmetry.

Diagonals of a parallelogram divide it into two pairs of symmetric triangles with respect to the intersection point of its diagonals. The parallelogram is symmetric to itself about this point.