MATH 7: HANDOUT 10
POKER PROBABILITIES

Poker Probabilities

In the game of poker, a player is dealt five cards from a regular deck with 4 suits (♠, ♣, ♢, ♡) with card values in the following order: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A. We calculated probabilities of the following combinations:

Royal Flush: 10, J, Q, K, A of any suit (Example: 10♡, J♡, Q♡, K♡, A♡)
There are only 4 of them.

Straight Flush: Five cards in a row of the same suit (Example: 6♡, 7♡, 8♡, 9♡, 10♡)
Each of these can start from any card from A to 9, and be in each of the for suits: $9 \times 4 = 36$. Notice that we excluded royal flushes from out computation (if we start with 10, we get a Royal Flush).

Four of a kind: Four cards of the same value, and one other random card (Example: K♡, K♠, K♢, K♣, 2♣)
Which card $13\times$ Which other value $12\times$ Which other suit $4 = 13 \cdot 12 \cdot 4$.

Full House: Three cards of the same value, and two cards of the same value (Example: K♡, K♠, K♢, 4♠, 4♣)
Which card for 3 $13\times$ Which three suits $\binom{4}{3}\times$ Which card for a pair $12\times$ Which two suits $\binom{4}{2} = 13\binom{4}{3}\cdot 12\binom{4}{2}$.

Flush: Five cards of the same suit, not in order (Example: 3♡, 6♡, 8♡, J♡, A♡)
Which suit $4\times$ Which five cards $\binom{13}{5} = 4\binom{13}{5}$. We also need to exclude Royal Flushes and Straight Flushes, so the total is $4\binom{13}{5} - 40$.

Straight: Five cards in order, possibly of different suits (Example: 5♡, 6♠, 7♢, 8♠, 9♣)
Which card to start from (anything from A to 10) $10\times$ Five suits $4^5 = 10 \cdot 4^5$. From here we also need to exclude Royal Flushes and Straight Flushes, so the final answer if $10 \cdot 4^5 - 40$.

Triple: Three cards of the same value, and two other random cards (Example: K♡, K♠, K♢, 4♠, 2♣)
Which card $\binom{13}{1}\times$ Which three suits $\binom{4}{3}\times$ Which two other values $\binom{12}{2}\times$ Which two suits for these two random card $4^2 = \binom{13}{1}\binom{4}{3}\binom{12}{2}4^2$.

Two pairs: Two cards of the same value, two cards of the same value, and a random card (Example: K♡, K♠, 10♢, 10♠, 4♣)
Which two cards $\binom{13}{2}\times$ Two suits for each of pair $\binom{4}{2}^2\times$ Remaining value $11\times$ Remaining suit $4 = \binom{13}{2}\binom{4}{2}^2 \cdot 11 \cdot 4$.

Pair: Two cards of the same value, and three other random cards (Example: K♡, K♠, Q♢, 4♠, 2♣)
Which card $\binom{13}{1}\times$ Which two suits $\binom{4}{2}\times$ Which three other values $\binom{12}{3}\times$ Which three suits for these three random card $4^3 = \binom{13}{1}\binom{4}{2}\binom{12}{3}4^3$.

To calculate probabilities of each of these combinations, we have to divide the counts above by the total number of poker hands, which is $\binom{52}{5}$. The table below gives the probabilities and odds:

Combination	Count	Probability	Odds
Royal Flush	4	0.000154%	1 : 649,740
Straight Flush	36	0.00139%	1 : 72,192
Four of a Kind	$13 \cdot 12 \cdot 4$	0.024%	1 : 4,165
Full House	$13\binom{4}{3} \cdot 12\binom{4}{2}$	0.1441%	1 : 693
Flush	$4\binom{13}{5} - 40$	0.1965%	1 : 508
Straight	$10 \cdot 4^5 - 40$	0.3925%	1 : 254
Triple	$\binom{13}{1}\binom{4}{3}\binom{12}{2}4^2$	2.1128%	1 : 46.3
Two Pairs	$\binom{13}{2}\binom{4}{2}^2 \cdot 11 \cdot 4$	4.7539%	1 : 20
Pair	$\binom{13}{1}\binom{4}{2}\binom{12}{3}4^3$	42.2569%	1 : 1.37
Nothing		50.1177%	1 : 0.995