# Math 6: Homework 2.11

## Inequalities

Today we discussed inequalities and their solutions, discussing how one solves inequalities involving negative numbers. The rule here is

$$a < b \Leftrightarrow (-a) > (-b)$$

(note that the sign of inequality is reversed!). For example, $3 < 5$, but $-3 > -5$.

More generally, if we multiply or divide both sides of an inequality by a negative number, we need to change the sign of inequality, replacing $<$ by $>$ and vice versa. For example, to solve $-3x > -6$, we divide both sides by $-3$ and change $>$ to $<$, giving $x < 2$.

### Products
The following rules are frequently used when dealing with equations or inequalities where the left-hand side is a product of two factors:
- A product of two numbers is zero if and only if one of them is zero:

$$ab = 0 \Leftrightarrow (a = 0) \ \text{ OR } \ (b = 0)$$

- A product of two numbers is positive if and only if both numbers are positive or both numbers are negative:

$$ab > 0 \Leftrightarrow (a > 0 \ \text{ AND } \ b > 0) \ \text{ OR } \ (a < 0 \ \text{ AND } \ b < 0)$$

- A product of two numbers is negative if and only if one of numbers is positive and the other one is negative:

$$ab < 0 \Leftrightarrow (a > 0 \ \text{ AND } \ b < 0) \ \text{ OR } \ (a < 0 \ \text{ AND } \ b > 0)$$

1) Solve the following inequalities, draw solution on the number line
a. $2x + 1 < 8x + 7$
b. $1 + 5x < 3x$
c. $x - 1 < x - 7$

2) Solve the following equations and inequalities:
a. $x(x + 1) < 0$

b. $\dfrac{1}{x} > 2$

c. $x^2 - 9 = 0$

d. $\dfrac{x}{x+1} > 1$

[**Hints**: In (b), multiple the inequality by x. You need to be careful, whether x is positive or negative. In (d), consider separately two cases: x + 1 > 0 and x + 1 < 0.]

Write the equations and inequalities for the following word problems and solve them:

3) Bob has $300 in savings at the beginning of the summer. He needs at least $100 at the end of it. He uses $30 per week for snacks, bus, and movie tickets. For how many weeks at most can Bob spend his money?

4) One pool contains 100 gallons of water, and the second - 150 gallons of water. Every hour 15 gallons of water are poured into the first pool, and 5 gallons into the second. At what point there will be more water in the first pool than in the second?

5) The first car plant produces no more than 950 cars per day. The second plant had 95% of the productivity of the first one. But after adding an additional line its production of cars increased by 23% compared to the first plant, and now it can produce more than 1000 units per day. How many cars did each plant produce per day before the reconstruction of the second plant?

6) In the figure below, each symbol stands for a number. The sum of numbers in each column or row is written next to the column or row—except for the second column, where the sum is not known. Can you find this missing sum?