

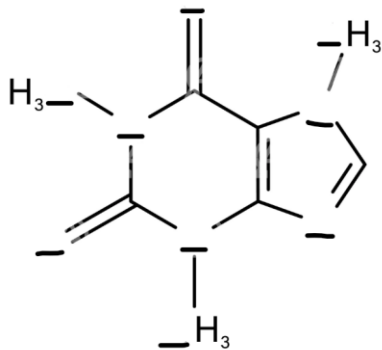
## Math 6: Fall 2023 Math Battle

- If you roll two dice:
  - What is the probability that the sum of the numbers will be larger or equal than 9? *(1 point)*
  - What is the probability that the sum of the numbers will be smaller or equal than 5? *(1 point)*
  - What is the probability that the sum of numbers will be larger than five and smaller than 9? *(1 point)*
- In a regular poker game, players are handed 5 cards each randomly from the deck. What is the probability that you get four of a kind? Four of a kind means that you get the same number from each kind. In poker, it does not matter the order in which you get the cards. Hint: First, determine in how many different ways you could make a four of a kind. Then, divide this by the total number of hands that could be dealt. *(3 points)*
- You meet two inhabitants: Sally and Zippy. Sally claims, 'I and Zippy are not the same.' Zippy says, 'Of I and Sally, exactly one is a knight.' Can you determine who is a knight and who is a knave? *(1 point, x2 points if you solve it with truth table)*
- On the island of Knights and Knaves, you meet three inhabitants: Bob, Mel and Peggy. Bob says that it's not true that Peggy is a knave. Mel says that Peggy is a knight or Bob is a knave. Peggy claims, "Both I am a knight and Bob is a knave." Can you determine who is a knight and who is a knave? *(2 points)*
- Recall the logical operation NAND, which is defined by the following truth table:

<i>A</i>	<i>B</i>	<i>A NAND B</i>
T	T	F
T	F	T
F	T	T
F	F	T

Show that any logical operation using NOT, AND, or OR can be rewritten in terms of the NAND operation. *(6 points = 2 for each of 3 operations)*
- I have three boxes. One is full of apples, another one is full of oranges, and the third one has a mixture of apples and oranges. The boxes are labelled "Apples", "Oranges", and "Mixed"; however, the three labels are all incorrect. You can choose any box to take one fruit out of it. Based on the result from that can you determine which box is which? Hint: let's look at the mixed box first, explain the logic. *(2 points)*

7. How many paths can you form on a grid with 2 rows and 5 columns if you start at the lower left corner and finish at the upper right corner? You can go only on the lines of the grid and you can only go to the right or up. (2 points)
8. Imagine that there are 20 points in the plane. Out of the 20 point, three of them are colinear, meaning that there is a line that contains the three points. How many different triangles could you do by picking three points out of the 20 possibilities? (2 points)
9. a. In a group of 60 people, 27 like tea and 42 like coffee and each person likes at least one of the two drinks. How many like both coffee and tea? (1 point)
- b. In a group of students, 25 study computer, 28 study Health, 20 study Mathematics, 9 study Computer only, 12 study Health only, 8 study Computer and Health only and 5 students study Health and Mathematics only.
- Draw a Venn diagram to illustrate the given information.
  - Find how many students study all the subjects at the same time?
  - How many students are there in total?
- (3 x 1 points)
10. A stubborn chemistry student refuses to learn the structure of the molecule caffeine. On their final exam, they are asked to fill in a diagram with which atom goes in which spot. The hydrogens are already filled in, but of the 9 remaining spots (marked as \_), the student has to fill in 3 carbon atoms, 4 nitrogen atoms, and 2 oxygen atoms. Assuming the student has no idea how chemical bonding works, how many ways is it to fill the diagram? (2 points)



11. I have several pieces of candy that I am putting into bags. I take one piece and put it into a bag, and call this bag 1. Then I take a second piece and put it in a bag, but I also put inside this bag a copy of bag 1 - so this bag has two objects in it: one piece of candy, and one bag with one piece of candy in it; I call this bag 2. Then I make bag 3 by putting inside it one piece of candy, a copy of bag 1, and a copy of bag 2. I do this again for bag 4 - bag 4 has in it a piece of candy, a copy of bag 1, a copy of bag 2, and a copy of bag 3. If I were to unpack bag 4 to get all the pieces of candy out, how many bags would I pull out in total? (2 points)