MATH 6: ASSIGNMENT 10
DECEMBER 10, 2023

Factorials and permutations

If we are choosing k objects from a collection of n objects so that

a) order matters and

b) no repetitions allowed (i.e. you can't pick up the same object twice),

then there are

$$P = n(n-1)(n-2)\dots(n-k+1) \qquad (k\ factors)$$

ways to do it.

If we take k = n, it means that we are selecting one by one all n objects — so this gives the number of possible ways to order n objects:

$$n! = n(n-1)\dots 2 \cdot 1$$

(reads n factorial).

For example: there are 52! ways to mix the cards in the usual card deck.

Note that the number n! grow very fast:

$2! = 2$
$3! = 6$
$4! = 4 \cdot 3 \cdot 2 = 24$
$5! = 120$
$6! = 620$

In all the problems that ask you to compute something, it suffices to write an expression for the answer, e.g., $\frac{1}{2^{11}}$; it is not necessary to perform the multiplication.

Algebraic Identities, we have been deriving pretty much every other class.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

HOMEWORK

1. About 1/6 of Americans have blue eyes. If we choose 10 people at random, what is the probability that all of them have blue eyes? that none has blue eyes? that at least one has blue eyes?

2. You have 6 different books on the shelf.
   a) In how many ways you can arrange them?
   b) What if all the books are identical?
   c) What if 5 books are identical?
   d) What if you have 3 books of one kind and 3 books of another kind?

3. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?

4. A puzzle consists of 9 small square pieces which must be put together to form a 3 x 3 square so that the pattern matches (this kind of puzzle is actually quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination a second, how long would it take you to try them all?

5. 10 people must form a circle for some dance. In how many ways can they do this?

6. Simplify:

(a) $\left(\frac{5a^2b^5}{4a^3b^3}\right)^3 =$ (b) $\left(2z^2 \cdot 3z^3 \cdot z\right)^2 =$ (c) $\frac{(-ab)^8}{(ab)^2} =$

(d) $\left(\frac{3ab^3}{15b}\right)^2 \cdot \frac{75c}{a^2b^6} =$ (e) $\left(\frac{3a^5b^2}{21ab}\right)^2 \cdot \frac{7^4}{a^{16}b^2} =$ f) $\frac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} =$

7. Open parentheses, simplify, use the knowledge of algebraic identities:
   a. $(2x - \frac{1}{2x})^2 =$
   b. $(\sqrt{a} + \frac{1}{2\sqrt{a}})^2 =$

8. At a fair, they offer you the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win a stuffed toy (worth about $5). Unfortunately, it is practically impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.
   a. If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
   b. Same questions for five balls.
   c. * They charge you 2 dollars for 3 balls, or 3 dollars for 5 balls. Which is a better deal? [*Considering only from the point of view of the chances of winning, not the fun you are getting*]