MATH 6
ASSIGNMENT 6: COMPUTER LOGIC

OCTOBER 29, 2023

Basic logic operations:

- NOT A: true if A is false, false if A is true
- A AND B: true if both A and B are true, false otherwise
- A OR B: true if at least one of A and B is true, false otherwise
- A XOR B: true if exactly one of A and B is true, false otherwise

Operation $\Rightarrow$ (reads "implies", or "if A then B"). Here are some of the more important rules:

- $A \Rightarrow B$ and $B \Rightarrow A$ are not equivalent: it is possible that one statement is true and the other is false.
- Contrapositive rule: $A \Rightarrow B$ is equivalent to $(\text{NOT } B) \Rightarrow (\text{NOT } A)$.

This construction is very useful in deducing new results from known ones. Here are some of the rules:

- Given $A \Rightarrow B$ and $B \Rightarrow C$, we can conclude $A \Rightarrow C$
- Given $A \Rightarrow B$ and NOT B, we can conclude NOT A
- Given $A \Rightarrow$ False, we can conclude NOT A (proof by contradiction)

LOGIC GATES AND COMPUTER CHIPS

Computer chips are using logical operations: each of the inputs and outputs can have voltage 0 or some positive voltage. The usual convention is

- Positive voltage=true
- Zero voltage=false

1. Let us consider a new logical operation, called NAND, which is defined by the following truth table:

A	B	A NAND B
T	T	F
T	F	T
F	T	T
F	F	T

(a) Show that A NAND B is equivalent to NOT$(A$ AND $B)$ (this explains the name: NAND is short for "not and").

(b) Show that A NAND A is equivalent to NOT A.

(c) Write the truth table for $(A$ NAND $B)$ NAND$(A$ NAND $B)$.

(d) Write the truth table for $(A$ NAND $A)$ NAND$(B$ NAND $B)$.

(e) Show that any logical formula which can be written using AND, OR, NOT can also be written using only NAND.

HOMEWORK

In problems 1–3, you need to a)write the obvious conclusion from given statements; and b)justify the conclusion, by writing a chain of arguments which leads to it. It may help to write the given statements and conclusion by logical formulas (denoting the statements which are used by letters $A, B, \ldots$ connected by logical operations OR, AND, $\Rightarrow, \ldots$).

1. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,...

2. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore, ...

3. Here is another one of Lewis Carrol's puzzles.
All hummingbirds are richly colored..
No large birds live on honey.
Birds that do not live on honey are dull in color.
Therefore,...

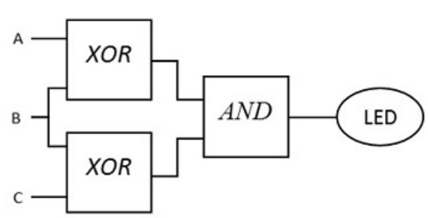
4. Simplify the following expressions.
(a) $\dfrac{6^5 \cdot 2^5}{3^5 \cdot 2^2} =$
(b) $(5^3)^3 \cdot 5^{-6} =$
(c) $(7^2 \cdot 7^3)^2 =$
(d) $(2a-3)(3a+4)=$

5. Prove the contrapositive rule that $A \implies B$ is equivalent to $(\text{NOT } B) \implies (\text{NOT } A)$ by building their truth tables (thus, "If you do not see my mirrors, then I do not see you" is equivalent to "If I see you, then you see my mirrors").

6. Show that $A \text{ XOR } B$ is the same as $(\text{NOT } A) \text{ AND } B \text{ OR } A \text{ AND}(\text{NOT } B)$

7. A particular musical elephant enjoys dancing, but only if it is wearing purple. Observing this elephant, I take the following notes:
D = *the elephant is dancing*
P = *the elephant is wearing purple*
The elephant dances only when wearing purple. It sometimes naps, no matter the color it is wearing.
From this, conclude whether the following statements are true or false:
(a) $D \implies P$
(b) $P \implies D$

8. * The diagram below shows some circuit constructed of 3 logical chips (each with two inputs and one output; we draw them so that the inputs are on the left and the output, on the right). Can you determine for which values of inputs the LED will light up? [Hint: this is the same



a writing a truth table for some formula....]

Note: the wires connecting each of the chips and LED to the power source are not shown.