MATH 5 — PREREQUISITES FOR MATH 6

1. Program

- Algebraic expressions. Commutativity, associativity, distributivity.
- Equations. Solving word problems with equations.
- Powers of 2.
- Binary numbers.
- Powers. Negative powers. Scientific notation.
- $a^2 - b^2 = (a - b)(a + b)$
- Square roots.
- Pythagorean theorem.
- Basic probability theory, basic combinatorics.
- Geometry: parallel lines and angles (alternate interior, alternate exterior, corresponding).
- Parallelogram, various definitions, properties.
- Congruence tests for triangles (SAS, ASA, SSS).
- Isosceles triangle. Median, bisector, height.
- Trapezoid. Its midline. Area.

2. PROBLEMS

1. Rewrite each of the expressions below in the simplest possible form, by collecting the like terms if possible.

   (a) $2x + 7 + 5x + 2 + 3x$
   (b) $3x + 9 + 5xy + 2xy + 3$
   (c) $3(2x - 1) + x$
   (d) $2a(a - 2) - a(a - 1)$
   (e) $(2x - 1)(x + 1)$

2. An apple cost 9 cents, and an orange 15 cents. Elena bought some apples and oranges, 20 fruit in all, and paid \$2.64. How many apples and how many oranges did she buy?

3. A boy had a bag of apples. He gave 1/2 of them to his parents, 1/5 to his brother, 1/4 to his sister and the last apple he ate himself. How many apples did he originally have?

4. If you take half my age and add 7, you get my age 13 years ago. How old am I?

5. Simplify the following expressions
   (a) $x + 4(1 - x)$ (b) $2 + 5x - 4(3 - x)$ (c) $5(x - 1) - 3(2x + 1)$

6. ~~If you take half my age and add 7, you get my age 13 years ago. How old am I?~~

7. Two secretaries, Barbara and Mary, need to type a 100 page document. Barbara can type it in 4 hours; Mary types slower, so it would take her 5 hours to do this. How fast can they type it together if they divide the work between two of them in the most efficient way?

8. Find the sum $1 + 2 + 4 + \cdots + 2^n$ (the answer, of course, will depend on n). [Hint: first try computing it for several small values of n: find $1 + 2$, then $1 + 2 + 4$, then $1 + 2 + 4 + 8$. See if you can notice a pattern. After this, try formulating a general rule.]

9. Convert the decimal numbers to binary:
     9, 12, 24, 38, 45

10. Convert the following binary numbers to decimal:
     101, 1001, 10110, 11011, 10101

11. Compute $110101_b + 111011_b$ without converting numbers to decimal form.