MATH 5: HANDOUT 16
BEGINNING PROBABILITY – 1.

BEGINNING PROBABILITY THEORY

We will be talking about "tests" (such as tossing a coin, rolling a die, drawing a card, etc), each of which can result in one of several possible outcomes (e.g., rolling a die can give numbers 1 through 6). If there are n possible outcomes, and they are all equally likely, then probability of getting any given one is exactly $1/n$; for example, probability of rolling a 3 on a die is $1/6$.

In most cases, we will be interested in probability of something that can be obtained in more than one way. For example, if we ask what is the probability of rolling an even number on a die, then there are 3 ways to get it: by rolling 2, 4, or 6. Each of these outcomes has probability $1/6$, so the probability of getting one of them is $1/6 + 1/6 + 1/6 = 3/6 = 1/2$. In general, if we ask what is the probability of getting one of a certain collection A of outcomes, then the answer is given by

$$P(A) = \frac{\text{number of outcomes giving } A}{\text{total number of possible outcomes}}$$

ADDITION RULE

Suppose we are drawing a card from the deck of 52 cards and ask what is the probability of getting either queen or king. Since there are 4 queens and 4 kings, which makes it 8 cards total, we can write

$$P(\text{queen or king}) = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$$

We can also write it as follows:

$$P(\text{queen or king}) = \frac{4+4}{52} = \frac{4}{52} + \frac{4}{52} = P(\text{queen}) + P(\text{king})$$

In general, we have the following rule:

$$P(A \text{ or } B) = P(A) + P(B)$$

if A and B can't happen together (see below).

Note that this rule only applies if A and B do not happen together. For example, there are 26 red cards in the deck, so probability of drawing a red card is $\frac{26}{52} = \frac{1}{2}$. However, if we need to get a red card or a queen, then using addition formula would give $\frac{26}{52} + \frac{4}{52} = \frac{30}{52}$, which is incorrect: this way, we have counted red queens twice. Correct answer is $\frac{28}{52}$: 26 red cards plus two black queens (no need to counte red queens –they have already been counted).

COMPLEMENT RULE

$$P(\text{not } A) = 1 - P(A)$$

For example, probability of drawing a queen from a deck of cards is $\frac{1}{13}$; thus, the probability of drawing something other than a queen is $1 - \frac{1}{13} = \frac{12}{13}$

Classwork

1. What's the probability that the product of the two is odd?

Solution: For the product to be odd, it should be odd times odd. There are $\frac{1}{2}$ the numbers that are odd, so $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ chances.

2. If we roll two dice, what is the probability that the product of two numbers is a multiple of 3?

Solution: For the product of two numbers 1–6 to be a multiple of 3, you can roll a 3 or 6 on first roll and then get anything else. These are 6 outcomes for, say, 3. If you roll either on the second roll, it is good except that we would double count the case where the first roll gave a 3 or a 6. So only 4 additional outcomes. So total number of favorable outcomes is $6 + 6 + 4 + 4 = 20$ out of a total of 36 outcomes. Thus the probability is $\frac{20}{36} = \frac{5}{9}$.

Easier way of doing this: As we know, 3 or 6 on any dice would give product as multiple of 3. So for product not to be multiple of 3, we have to get from (1,2,4,5) on both dice. That probability is $\frac{4}{6} \times \frac{4}{6} = \frac{4}{9}$. Thus probability of product being 3 is $1 - \frac{4}{9} = \frac{5}{9}$.

3. Suppose we have a box of 500 candy of different colors and sizes. We know that there are 100 large ones and 400 small ones; we also know that there are 70 red ones, 11 of which are large. From this information, can you compute the probability that a randomly chosen candy will be either red or large (or both)?

Solution: The possible events are 70 (for red) + 89 (100-11 to not double count) large. Thus the probability of either red or large is $\frac{159}{500}$.