MATH 5: SQUARE ROOT. PYTHAGOREAN THEOREM.

SQUARE ROOTS

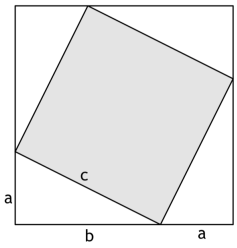
Square root of a is a number whose square is equal to a. For example: square root of 25 is 5, because $5^2 = 25$.

- Notation: square root of number a is commonly denoted $\sqrt{a}$.
- $\sqrt{ab} = \sqrt{a}\sqrt{b}$, but $\sqrt{a+b}$ is **not** equal to $\sqrt{a} + \sqrt{b}$.

Pythagorean theorem: In a right triangle with legs a, b and hypotenuse c, one has

$$a^2 + b^2 = c^2 \qquad \text{ot} \qquad c = \sqrt{a^2 + b^2}.$$

Proof: Consider the following picture:



In this square, the total area is

$$(a+b) \times (a+b) = a \times (a+b) + b \times (a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

On the other hand, the area of each triangle is $\frac{1}{2}ab$, and the area of shaded square is c^2. Thus, we get

$$a^2 + 2ab + b^2 = 4 \times \tfrac{1}{2}ab + c^2$$

which gives $a^2 + b^2 = c^2$. □

For example, in a square with side 1, the diagonal has length $\sqrt{2}$.

It is possible — but not easy — to find a right triangle where all sides are whole numbers. The easiest such triangle is the triangle with sides $3, 4, 5$.

POWER $\frac{1}{2}$

We know how to raise numbers into whole powers:

$$a^n = a \times \cdots \times a.$$

But what is $a^{\frac{1}{2}}$?

Example: Let's try to figure out what $4^{\frac{1}{2}}$ is:

$$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^{\frac{1}{2}+\frac{1}{2}} = 4^1 = 4.$$

We can see that $4^{\frac{1}{2}}$ must be a number, such that if we multiply it by itself, we get 4. But this is just a square root of 4! So, we get:

$$4^{\frac{1}{2}} = \sqrt{4}.$$

In general, this is also true:

$$a^{\frac{1}{2}} = \sqrt{a}.$$

HOMEWORK

1. Find the following square roots. If you can not find the number exactly, at least say between which two whole numbers the answer is, e.g., between 5 and 6.

(a) $\sqrt{16}$

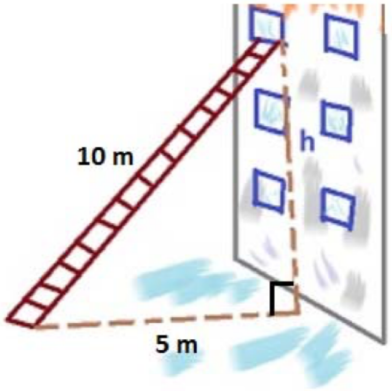
(b) $\sqrt{81}$

(c) $\sqrt{10,000}$

(d) $\sqrt{10^8}$

(e) $\sqrt{50}$

2. Find the distance from the window to the ground h. Hint: use Pythagorean theorem.



3. If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?

4. Can you find a right triangle where all sides are whole numbers and the hypotenuse is 13?

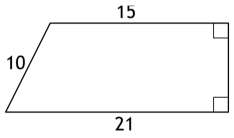
5. Find

(a) $\sqrt{2^6 \times 7^2}$

(b) $\sqrt{\frac{1}{16}}$

(c) $\sqrt{\frac{4}{9}}$

6. Find the height and area of the figure below. Lengths of three sides are given; the two marked angles are right angles.



7. The side of an equilateral triangle is 1 m. Find its height and the area.

8. Open parenthesis and simplify

(a) $3(a-5)-2(2a-9)=$

(b) $12x-3x(x+4)=$

(c) $5x-5(7-a+x)=$

(d) $2a(a-2)-a(a-1)=$