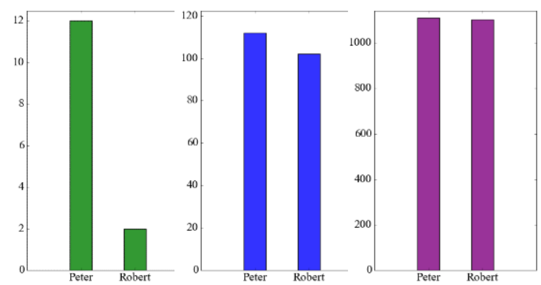
Classwork 6.

Ratio and proportions.

Peter has 10 dollars more than Robert. Is this a big difference? How we can compare the amount of money they have?
Take a look at the table

Peter	\$12	\$112	\$1112
Robert	\$2	\$102	\$1102

In all these cases the absolute difference is the same, but in the first case Peter has 6 times as much as Robert, in the last situation they both have almost the same amount of money. The ratios of the amount of Peter's money and Robert's money are.



$$\frac{12}{2}; \qquad \frac{112}{102}; \qquad \frac{1112}{1102};$$

The amount of money Peter and Robert have in the first case is 12 and 2 dollars and the ratio is $\frac{12}{2} = 6$, or 6:1, or 6 to 1.

Example1: (**it's not a real recipe**) The ratio of water and lemon juice in lemonade is 4 to 1. What does it mean? In means that for each part of lemon juice we need to add 4 parts of water, or the volume of lemon juice and water should have the same ration as 1 to 4:

$$\frac{volume\ of\ juice}{volume\ of\ water} = \frac{1}{4}$$

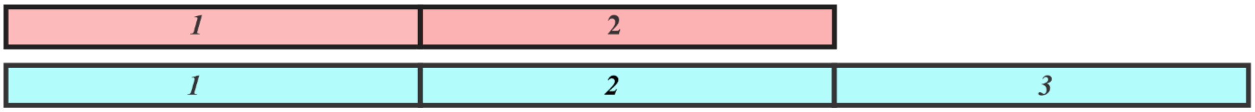
How much of lemon juice and water we need to prepare 1 l. of lemonade? 1.5 l.?
Total number of volume "units" is $5 = 4 + 1$, for total volume, $\frac{1}{5}$ is juice, and $\frac{4}{5}$ is water.
We want to have sweet lemonade and we add sugar. The ratio of water, lemon juice and sugar is 4:1:0.5 (or it can be rephrased as 8:2:1). For each part of sugar, we will use 2 parts of lemon juice, and 8 parts of water.

We can write the ratio of two numbers in the several ways:

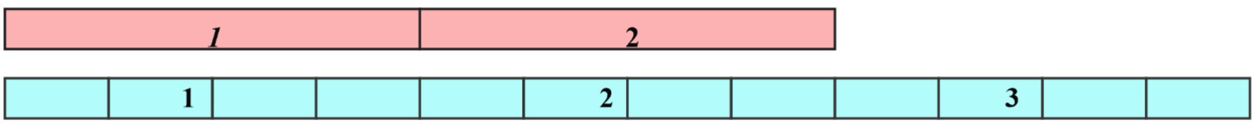
$$a\ to\ b, \qquad a\!:b, \qquad \frac{a}{b}$$

Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold 3/4 of the blue balloons, 1/2 of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?

Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:

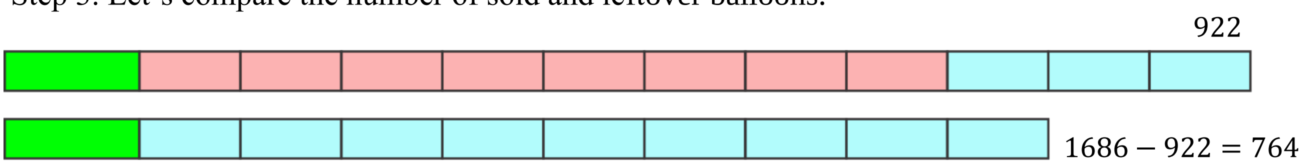


We took as "unit" a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more than number of red balloons (or three times as much as a half of the red ones)



Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can't divide 3 "units" into 4 parts, without getting fractions. So, let's find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.

Step 3. Let's compare the number of sold and leftover balloons.



Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 small "units" of blue balloons are $922 - 764 = 158$, or one such "unit" is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of green ones is $1686 - (632 + 948) = 106$. Can we solve the problem by writing equations?

Let's try.

$G + B + R = 1686$

$3R = 2B$

$\frac{1}{2}G + R + \frac{1}{4}B = 922$

$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$

$$R - \frac{1}{2}B = 158$$

$$\frac{2}{3}B - \frac{1}{2}B = 158 \quad \Rightarrow \quad \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \quad \Rightarrow \quad B = 6 \cdot 158$$

To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3:2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$

Two ratios which are equal form a proportion.
Proportions have several interesting features.

inside terms

$$\frac{a}{b} = \frac{c}{d} \quad or \quad a:b = c:d$$

outside terms

1. The products of inside and outside terms are equal.
$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c$$
It can be easily shown:
$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{adb}{b} = \frac{cdb}{d} \Leftrightarrow ad = cb$$
2. Also, two inverse ratios are equal:
$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{d}{c}$$

Indeed:
$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ac} = \frac{bc}{ac} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$

3. Two outside terms can be switched:
$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{d}{b} = \frac{c}{a}$$
$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ab} = \frac{bc}{ab} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$
4. Two inside terms can be switched as well.
$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{c} = \frac{b}{d}$$
5. Also, several other new proportion can be created.
$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a \pm b}{b} = \frac{c \pm d}{d}$$
(The sign $\pm$ is used to show that both, addition and subtraction, can be used)
Let's prove one of the statements:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{b} + 1 = \frac{c}{d} + 1 \quad \Leftrightarrow \quad \frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d} \quad \Leftrightarrow \quad \frac{a+b}{b} = \frac{c+d}{d}$$

6. Another proportion:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a+c}{b+d} = \frac{c}{d} = \frac{a}{b}$$

It can be proved as follow:

$$\frac{a+c}{b+d} = \frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}$$

We know form (4) that

$$\frac{c}{a} = \frac{d}{b}$$

$$\frac{a+c}{b+d} = \frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)} = \frac{d}{b}$$

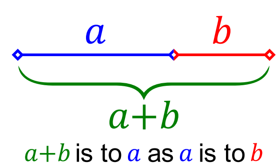
Going back to the jam problem above. We got the simple equation

$$\frac{3}{2} = \frac{27}{x}$$

It can be solved easily using the property of proportion

$$3x = 27 \cdot 2$$

$$x = \frac{27 \cdot 2}{3} = \frac{3 \cdot 9 \cdot 2}{3} = 18$$



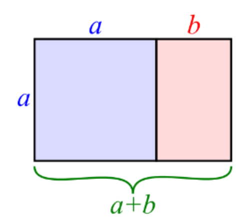
Famous ratios.

Let’s measure the circumference and the diameter of a circle.

$$\frac{l}{d} = \pi$$

Golden ratio

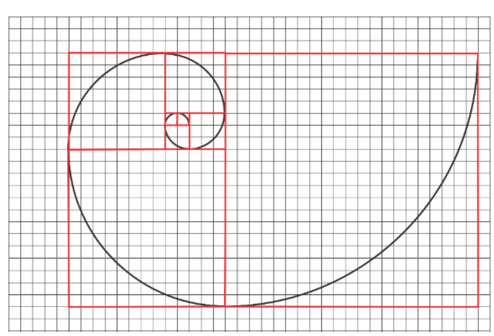
$$\frac{a+b}{a} = \frac{a}{b} \cong 1.618$$

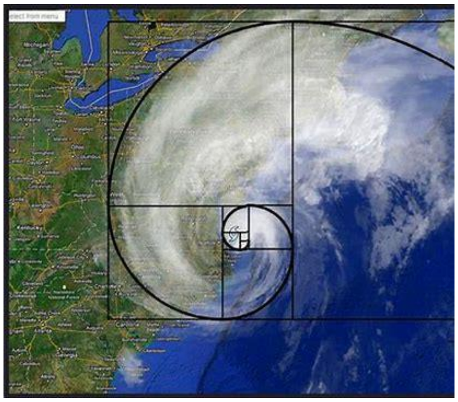


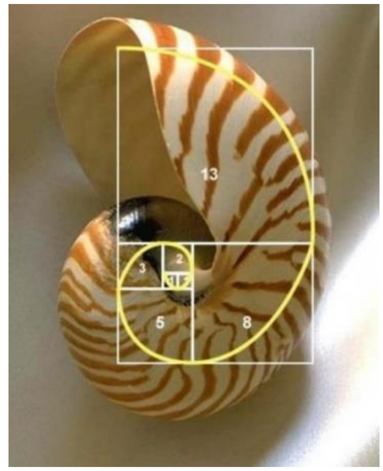
Fibonacci sequence:

$$1, 1, 2, 3, 5, 8 \ldots..$$

$$F_n = F_{n-1} + F_{n-2}$$







Exercises:

1. Mr. Robinson was paid $590 for a job that required 40 hours of work. At this rate, how much should he be paid for a job requiring 60 hours of work?

2. If two pounds of meat will serve 5 people, how many pounds will be needed to serve 13 people?

3. 6 oxen or 8 cows can graze a field in 28 days. How long would 9 oxen and 2 cows take to graze the same field?

4. In a dried fruit mix, there are 7parts of dried apples, 4 parts of dried pears and 5 parts of dried apricots. What is the weight (how many grams) of apples, pears, and apricots in the fruit mix, if the total weight of the mix is 1600g?

5. A merchant accidentally mixed candies of the first type (priced at $3 per pound) with candies of the second type (priced at $2 per pound). At what price should this mixture be sold to obtain the same total amount, given that it is known that initially the total cost of all candies of the first type was equal to the total cost of all candies of the second type?

6. The whole family drank a full cup of coffee with milk, with Robert drinking a quarter of all the milk and a sixth of all the coffee. How many people are in the family?

7. Solve the equations:

$a.\ \ \frac{6x}{25} = \frac{0.4}{0.15};\qquad b.\ \ 1\frac{1}{9} : (0.8y) = \frac{1}{7} : 3.6;\qquad c.\ \ \frac{1.25}{0.06} = \frac{z-6}{2.4};\qquad d.\ \ \frac{7}{2+t} = \frac{4.2}{t}$