

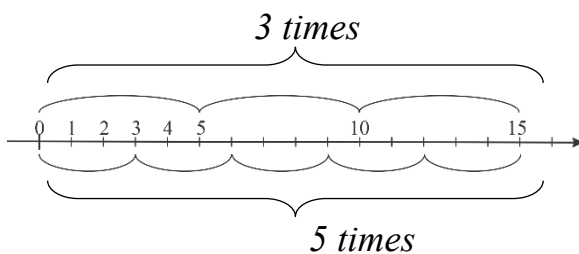
Math 4b. Homework 4.



Can we split 15 candies between kids equally?

We can divide equally 15 candies between 5 kids. Because 5 can fit into 15 exactly 3 times. 15 is called a **dividend**, 5 is called **divisor**, and the result 3 is called a **quotient**. When we are doing division of a number (the **dividend**) by a **divisor**, we are trying to find a number (a **quotient**), that produces the dividend if multiplied by the divisor.

We can also divide equally 15 candies between 3 kids. 3 can go into 15 exactly 5 times. We can say that 15 is **divisible by 3** and by 5.



If there is a number c , that $c \times b = a$, then we can say that $a \div b = c$, and a is **divisible by** b , b can be “fit” into a whole number of times.

We can also say that b is a **factor** of a , and c is also a **factor** of a , $a \div c = b$.

For example,

$$3 \times 5 = 15; \quad 15 \div 3 = 5, \quad 15 \div 5 = 3$$

$$\begin{array}{c}
 \text{factor} \quad \quad \quad \text{factor} \quad \quad \quad \text{product} \\
 \swarrow \quad \quad \quad \nwarrow \quad \quad \quad \swarrow \\
 5 \cdot 3 = 5 + 5 + 5 = 3 + 3 + 3 + 3 + 3 = 15
 \end{array}$$

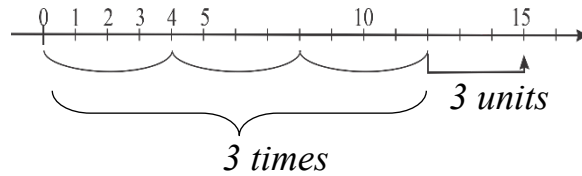
If there is no such number that the divisor enters the dividend several times, then we can say that this number is not divisible by the divisor. But in this case, we can use **division with a remainder**.

$$\begin{array}{c}
 a : b = c \\
 \swarrow \quad \uparrow \quad \nwarrow \\
 \text{dividend} \quad \text{divisor} \quad \text{quotient}
 \end{array}$$

$$\begin{array}{c}
 a = b \cdot c + r \\
 \swarrow \quad \uparrow \quad \nwarrow \quad \swarrow \\
 \text{dividend} \quad \text{divisor} \quad \text{quotient} \quad \text{remainder}
 \end{array}$$

We can't divide 15 candies equally between 4 kids. $15 \div 4$.

4 can't fully complete 15. It can fit into 12 three times, but there will be a little more left (3 to be exact). So, $15 \div 4 = 3R(3)$, or $15 = 4 \times 3 + 3$



For division of any natural number by another, we can now write:

$$a \div b = cR(r), \quad \text{or} \quad a = b \times c + r$$

If $r = 0$, number a is divisible by number b .

Divisibility rules.

Can we predict whether a given number is divisible by 2, 3, 4, 5 and so on? There exist the following divisibility rules.

1. A number **is divisible by 2** if and only if **its last digit is even or 0**.

Let's try to see why it's true.

Any natural number can be written in an extended form:

$$\dots + 100 \cdot (\text{number of hundreds}) + 10 \cdot (\text{number of tens}) \\ + \text{number of units}$$

If the last digit, number of units, is even, then each term has a common factor 2 and this factor can be factor out of the parenthesis, and number will have a factor 2, so, will be divisible by 2. If it's 0, we don't have any units, number is divisible by 10, and 10 is a product of 2 and 5.

2. A number **is divisible by 3** if and only **if sum of its digits is divisible by 3**.
3. A number **is divisible by 4** if and only if the **number formed by the last 2 digits is divisible by 4**.
4. A number **is divisible by 5** if and only if its **last digit is 5 or 0**.

Homework problems

1. There are 24 students in the class.

How they can be divided into equal teams?

What are the factors of 24?

2. Find all factors of numbers

- a. 6,
- b. 7,
- c. 14,
- d. 18,
- e. 70.

3. If we want to divide a number by 9, what numbers can we get as a remainder?

4. Fill in the empty cell in the table:

dividend	a	29		46	94
divisor	b	7	9		9
quotient	c	4	7	3	
remainder	r		5	1	4

Check the formula $a = b \cdot c + r$ for each number in the table.

5. Even or odd number will be the sum and the product of

- a. 2 odd numbers
- b. 2 even numbers
- c. 1 even and 1 odd number
- d. 1 odd and 1 even number

Can you explain why?

- 6.
- a. Will the following numbers be divisible by 2:
123457, 1029384756, 43567219874563157830
 - b. by 3
1347, 45632, 5637984265
 - c. by 5:
5635, 78530, 657932, 45879515
 - d. by 4
1216, 13944, 893057, 32348909824
7. Give an example of a number which gives the remainder 1 when divided by 2 and by 3.
8. Give an example of a number which gives the remainder 1 when divided by 2 and remainder 2 when divided by 3.
9. * If pencils are put into boxes, 8 pencils in a box, then 5 extra pencils will remain. If 6 pencils are put in each box, then there will also be 5 extra pencils. How many pencils are there if there are more than 50 but less than 100? (Hint: number of filled boxes in two cases will be different.)
10. Show how to cut the shape below into two parts that can be rearranged into a square without overlapping. The cut should go along the grid lines but does not have to be a straight line.

