Math 4a. Classwork 23.

If a number a in a power n is divided by the same number in a power m,

$$\frac{a^n}{a^m} = \frac{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}} = \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}\right) : \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}\right) = a^n : a^m = a^{n-m}$$

So, we can say that

$$a^{-1} = \frac{1}{a^1} = \frac{1}{a}; \qquad a^{-n} = \frac{1}{a^n};$$

Let's see how our decimal system of writing numbers works when we use the concept of exponent: $3456 = 1000 \cdot 3 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 6 = 103 \cdot 3 + 102 \cdot 4 + 101 \cdot 5 + 100 \cdot 6$

The value of a place of a digit is defined by a power of 10 multiplied by the digit. Very large numbers can be written using this system, as well as very small numbers.

$$0.3 = \frac{1}{10} \cdot 3 = 10^{-1} \cdot 3;$$

$$0.456 = \frac{1}{10} \cdot 4 + \frac{1}{100} \cdot 5 + \frac{1}{1000} \cdot 6 = 10^{-1} \cdot 4 + 10^{-2} \cdot 5 + 10^{-3} \cdot 6$$

1. Write the following expressions in a shorter way replacing product with power:

   Examples:

   $(-a) \cdot (-a) \cdot (-a) \cdot (-a) = (-a)^4,\quad 3m \cdot m \cdot m \cdot 2k \cdot k \cdot k \cdot k = 6m^3k^4$

   $a.\ (-y) \cdot (-y) \cdot (-y) \cdot (-y);$ $\qquad b.\ (-5m)(-5m) \cdot 2n \cdot 2n \cdot 2n;$

   $c.\ -y \cdot y \cdot y \cdot y;$ $\qquad d.\ -5m \cdot m \cdot 2n \cdot n \cdot n;$

   $e.\ (ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab);$ $\qquad f.\ p - q \cdot q \cdot q \cdot q \cdot q;$

   $g.\ a \cdot b \cdot b \cdot b \cdot b \cdot b;$ $\qquad h.\ (p - q) \cdot (p - q) \cdot (p - q);$

2. Simplify the expressions:

$a.\quad 2^4 + 2^4;$ $\quad b.\quad 2^m + 2^m;$ $\quad c.\quad 2^m \cdot 2^m;$

$d.\quad 3^2 + 3^2 + 3^2;$ $\quad e.\quad 3^k + 3^k + 3^k;$ $\quad f.\quad 3^k \cdot 3^k \cdot 3^k;$

3. What will be last digit of

$a.\ 2^{22};$ $\quad b.\ 3^{33};$ $\quad c.\ 4^{44};$ $\quad d.\ 5^{55};$ $\quad e.\ 6^{66};$ $\quad f.\ 7^{77};$

4. Compare:

Example: What is greater 31^{11} or 17^{14}?

We can see that $31 < 32 = 2^5$; $2^4 = 16 < 17$,

$$31^{11} < 32^{11} = (2^5)^{11} = 2^{55}$$

$$(17)^{14} > 16^{14} = (2^4)^{14} = 2^{56}$$

We can write the following:

$$31^{11} < 32^{11} = 2^{55} < 2^{56} = 16^{14} < (17)^{14}$$

$$31^{11} < 17^{14}$$

a. 127^{23} and 513^{18}

b. 9997^{10} and 100003^8

c. 5^{300} and 3^{500}

5. Write the following numbers as the power of base 10:

10, 100, 1000, 10000, 100000, 1000000

0.1, 0.01, 0.001, 0.0001, 0.00001, 0.000001

6. Reduce the fractions

$a.\ \frac{49^4 \cdot 7^5}{7^{12}};$ $\quad b.\ \frac{3^{10} \cdot 27}{81^3};$ $\quad c.\ \frac{125^3 \cdot 5^7}{5^{18}};$